AFNOR Standard NF 50-144-3

Evaluation of the Disjoint Block Method (DBM)

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The new standard AFNOR XP X50-144 « *Demonstration of the resistance to environmental conditions — Design and Carrying of environmental tests" Part 3: Application of the personalization approach in a mechanical environment*" published in January 2021 requires the use of the Disjoint Block Method (DBM) to assess the severity of vibratory environments and transform the measurements into specifications.

This document presents the main results of a study of this new DBM method, a priori interesting, carried out to evaluate on a few cases its applicability and its possible limits.

1- REMINDERS

The so-called "tailoring" approach has been implemented since the 1980s [GAM 86], then [NOR 14]. The test specification determination process can be divided into four main steps :

1. analysis of the lifecycle profile;
2. collection of data on the real environment;
3. synopsis of the data;
4. establishment of the test program.

The vibrational environment data (vibrations and shocks) collected during step 2 characterizing each event of each situation of the life profile of the material or equipment studied are the subject of a frequency analysis by calculation of power spectral densities (PSD) if theoretically possible, extreme response spectra (ERS) and fatigue damage spectra (FDS). The specifications are written from summaries involving envelopes of SRS, ERS and sums of FDS, with uncertainty coefficients (CG) and test factors (FE) intended to take into account the variability of the environment and the resistance of the materials as well as the low number of tests carried out to qualify the equipment during the tests.

It is recalled that the SRS gives the largest peak of the response of a linear mechanical system with a single degree of freedom (dof) as a function of its natural frequency, for a given value of its Q factor.

The ERS ("deterministic"), calculated from a vibration signal (random, sine, etc.), is the equivalent of the SRS. It can also be calculated from the PSD of a random vibration when its distribution of instantaneous values is Gaussian and in this case it gives the peak with a probability of 60% of being

exceeded. During the equivalences carried out to define a specification from these spectra, the calculation of the ERS of the random vibration specification is carried out with this same value, which in general does not pose any difficulty. However, we also know how to calculate from a PSD a spectrum of the same nature, but for a probability of up-crossing chosen a priori, for example of a few percent, called URS (response spectrum with up-crossing risk).

The FDS gives as for him the damage by fatigue created on a system with only one dof according to the natural frequency for a given Q factor and for a given value of the parameter b of the law of Basquin used to represent analytically the curve of Wöhler of the material of the part considered to be the most fragile in terms of fatigue in the material. The FDS is an average spectrum and we also know how to calculate a spectrum for any probability of up-crossing, the URS. These different spectra are called “classic” in the following paragraphs, to distinguish them from “DBM” spectra.

The calculations of all these spectra are therefore either carried out directly from a signal as a function of time, or from a PSD. In the latter case, the signal must be stationary and Gaussian. When this is not the case, several spectra are calculated from the signal on areas considered to be more or less stationary and statistics are made on these curves.

The development of the DBM was launched to overcome the lack of a method for analyzing non-stationary / non-Gaussian signals, and in particular to allow the calculation of an URS spectrum at risk of a given up-crossing for a given duration.

2- THE DBM PROCESS

The main steps in this process are as follows:

1- Splitting of the signal of duration Ts into Nb blocks of the same duration Tb.

For each point of the ERS (f_0, Q):

2- Calculation of the time response of the system of each of the Nb blocks

3- Calculation of the largest response peak of each block. We thus obtain a set of Nb largest peaks

4- Distribution of peaks in Nc classes according to the stationarity of the blocks

5- For each class, calculation of the parameters of several statistical laws for choosing the most appropriate law according to a certain criterion (Gumbel, GEV2 and 3, Weibull 2P and 3P, LN 2P, LN 3P).

6- For the distribution chosen, modification of the parameters of the law retained to take account of the duration of the vibration Tv (> Ts).

7- Product of the Nc distributions (Gumbel, EV2, WBN) having the parameters thus calculated.

8- Search for the variable (point of the ERS) that corresponds to a given risk of up-crossing

Duration extrapolation

From the distribution law of the identified response peaks, for example Gumbel's distribution, defined by:

$$P(Z_p) = \exp\left\{-\exp\left[-\frac{Z_p - \mu}{\beta}\right]\right\} \qquad [1]$$

we have, for N peaks, the probability:

$$P_N(Z_p) = \left(\exp\left\{-\exp\left[-\frac{Z_p - \mu}{\beta}\right]\right\}\right)^N \qquad [2]$$

From the value of the parameters and for peaks, we calculate the necessary relation between the parameters and to obtain the same probability for peaks:

$$\left(\exp\left\{-N_2\ \exp\left[-\frac{Z_p - \mu_2}{\beta_2}\right]\right\}\right) = \left(\exp\left\{-N_1\ \exp\left[-\frac{Z_p - \mu_1}{\beta_1}\right]\right\}\right) \qquad [3]$$

From where

$$\frac{\mu_2}{\beta_2} = \left(\frac{1}{\beta_2} - \frac{1}{\beta_1}\right) Z_p + \frac{\mu_1}{\beta_1} + \log\left(\frac{N_2}{N_1}\right) \qquad [4]$$

A single relationship to estimate two parameters. It can be shown that $\beta_2 = \beta_1$. Hence, by putting $\beta = \beta_2 = \beta_1$,

$$\mu_2 = \mu_1 + \beta \log(M) \qquad [5]$$

with $M = \frac{N_2}{N_1} = \frac{T_v}{T_s}$.

NOTE :

For Frechet's distribution, $\beta_2 = \frac{\beta_1}{M^k}$

$$P = exp\left\{-\left[1 - k\,\frac{Z_p - \mu}{\beta}\right]^{\frac{1}{k}}\right\} \qquad [6]$$

$k = k_2 = k_1$

$$\mu_2 = \mu_1 + \frac{\beta_1}{k} \frac{M^k - 1}{M^k} \qquad [7]$$

3- RESULTS OF THE EVALUATION

3.1. Comparison of DBM vs classical method results

Initially, it was considered useful to ensure that the results of the DBM are consistent with those of the classical method. This verification can only be carried out for stationary Gaussian signals, by comparison with the classic URS and the DBM URS. Most of the spectra calculated in the following examples were obtained from a random signal as a function of Gaussian time generated using a control bay from a PSD:

Power Spectral Density (PSD)	Signal (Run1)
White noise : 0.1 g^2/Hz, 1 Hz to 400 Hz Frequence step : 0.125 Hz (i.e. 3200 spectral lines)	Gaussian random signal Duration : 10 min Sampling frequency : 12,800 Hz

During the ASTELAB 2021 symposium [LEL 21], a comparison was presented between a classic URS calculated from this PSD and an URS DBM from the signal (Figure 1) calculated under the

ERS - URS calculation conditions
☐ Signal and PSD (Run1)
☐ Analysis bandwidth : 1 Hz à 1 kHz
☐ Resolution : 1/24th octave
☐ Q = 10, b = 8, K = C = 1
☐ Extrapolation time: 10,000 h
☐ Block duration Tb (DBM) : 1 sec

This slide shows 3 curves:

- the ERS calculated from the signal as a function of time (“deterministic” ERS, in blue),
- the URS calculated from the PSD of the signal, for a duration of 10000 h, in red,
- the DBM URS (green curve).

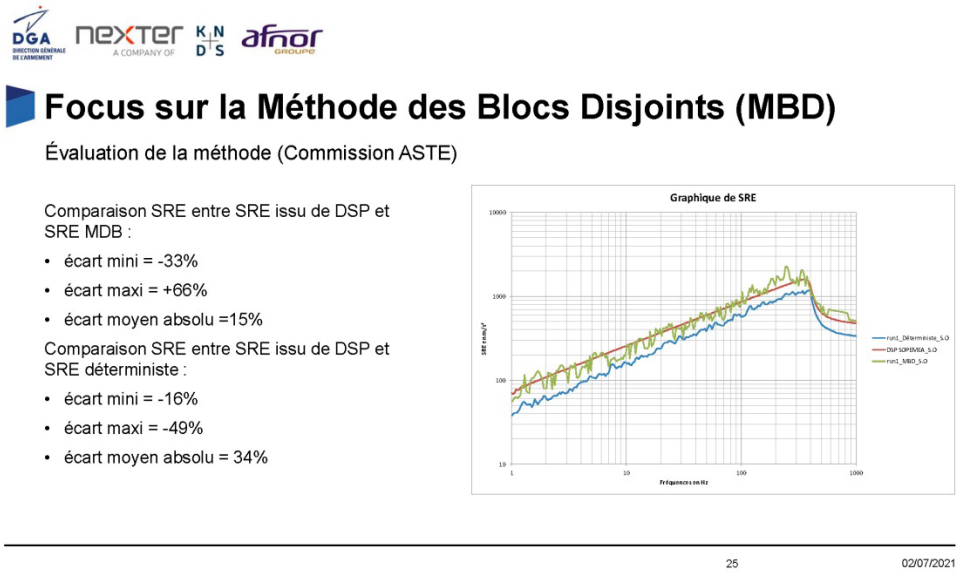

Figure 1. *Slide presented at ASTELAB 2021 [2].*

The differences between the URSs are best highlighted in linear axes (Figure 2).

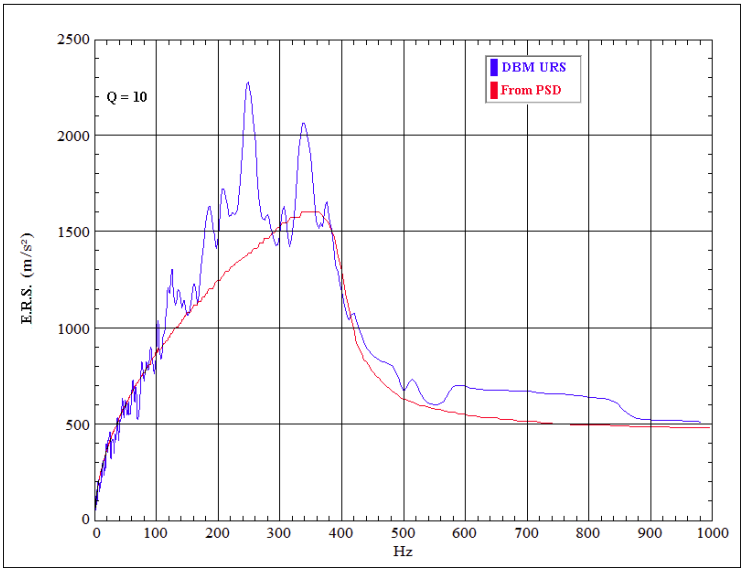

Figure 2. *Spectra of Figure 1plotted in linear axes*

NOTES :

1- *The "deterministic" ERS is not traced on this last figure, because it is analogous to an SRS, calculated in principle over the duration of the signal (600 s) and not for 10,000 h. so it cannot be used as a reference for a comparison, except to observe that it is much tormented than the DBM URS.*

This ERS is not representative of what the manufacturers concerned would have done in the classic approach on a data set with non-stationary and/or non-Gaussian signals: one could have imagined, for example, an approach of segmentation in stationary phases followed by 'a

calculation of the URS over 10,000 h from the PSDs of each segment and a calculation of the URS at n standard deviations (3 for example).

2- The use of the average of the absolute mean deviation over the frequency band was not relevant.

- *If it were to be used, the absolute mean deviation criterion would remain to be defined (unclear)*

- *The concept of average difference is questionable because what counts is the difference in amplitudes at a given frequency, a frequency which must be considered as random because we do not know in advance where it will be.*

The differences observed between the DBM and classic URS are as follows:

- Minimum difference : - 33%
- Maximum difference : + 66%
- Absolute mean difference : 15%

The maximum deviation of 66% is difficult to understand, knowing that the calculation of the URS from the PSD of a Gaussian signal is done statistically using Rice's law, rigorously established for signals of this type.

The difference between the URS from the PSD and the DBM URS can thus be significant. One cannot consider that the calculation of the DBM URS is in the state validated in a satisfactory way in the case of a Gaussian signal. This comparison is only feasible on Gaussian data. And as long as the registration on Gaussian data is not satisfactory, a fortiori it does not help to give confidence on non-Gaussian data.

The previous example shows that the DBM URS is a curve that presents many relatively large and rapid variations. The DBM method leads to results with many peaks whose cause remains undetermined to date.

The peaks are all the more important when :

- the extrapolation of the duration is greater,

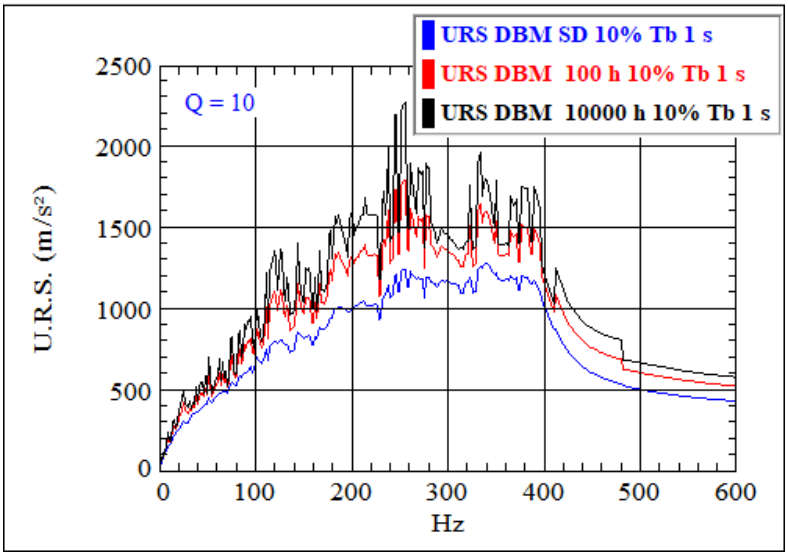

Figure 3. *DBM URS calculated from Run1 signal over signal duration (SD), over 100 h and 10000 h (600 points, Risk: 10%, Tb = 1 s)*

- the chosen up-crossing risk is lower,

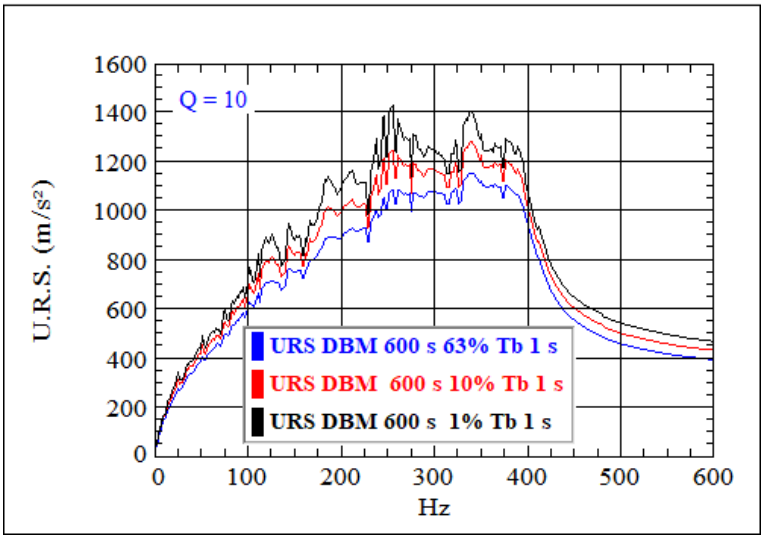

Figure 4. *DBM URS of Run1 calculated over the duration of the signal (600 s), Risk 63%, 10%, and 1% (Tb = 1 s)*

- the duration of the blocks is greater (cf. &3.3).

Among the attempts at explanation, the following have been mentioned :

- non-truly Gaussian signals which would be generated by the usual methods for generating these signals, in particular by the control bays. However, the characterization of these signals by plotting a Henry line does not confirm this hypothesis.

- a problem of identification of the statistical laws of the distribution of the peaks of the responses of each dof. When we look at the way in which the laws are chosen during the process, we

realize that this choice is made between values resulting from the tests which are very close and we can easily pass from one law to another between two close dof.

3.3. Influence of choice of block duration

During this work, the question of the influence of block duration on the reproducibility of DBM results was considered.

3.3.1. Comparison of DBM URS calculated for several Tb values

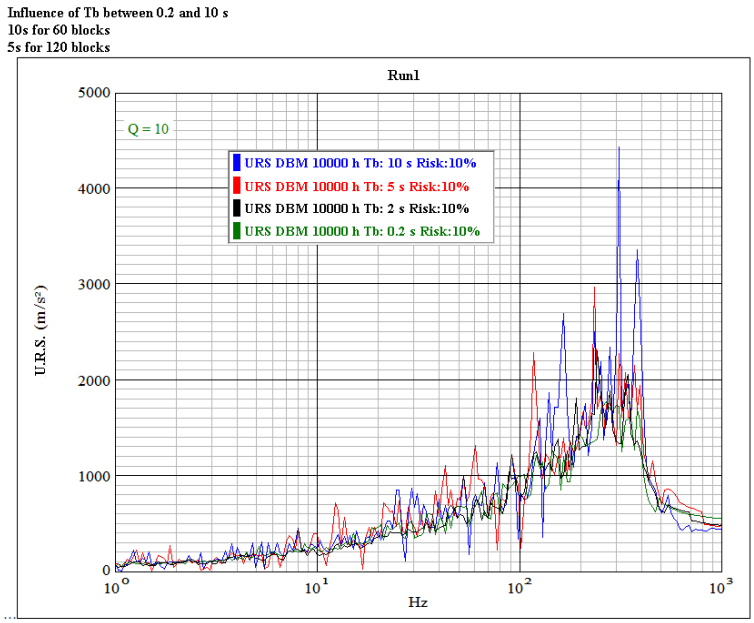

Figure 5. *DBM URS of Run1 for Tb =10 s, 5 s, 2 s and 0.2 s (Risk 10%, 10000 h)*

The graph in Figure 5 shows the DBM URSs of Run1, with extrapolation to 10,000 hours, for Tb ranging from 0.2 s to 10 s, which corresponds to Nb always greater than 40 (60 blocks for 10 s, 120 blocks for 5 s). It can be seen that the DBM URS peaks have a frequency position that depends on Tb, which is critical, and that the amplitudes of these peaks with Tb =10 s are more than 200% of the amplitude of those that we would obtain with a Tb equal to 0.2 s.

The block duration Tb is chosen a priori. The only information given in the Standard is the number of blocks (Nb) which must be greater than 40.

3.3.2. Variation of the maximum deviation as a function of Tb

Figure 6 shows the evolution of the difference in % between URS from PSD and DBM URS, from the Run1 signal for Tb values between 0.01 s and 10 s for an extrapolation period of 3000 h (Risk = 10%). The gap goes through a minimum at 40% and then goes up.

These variations can be explained by the number of peaks among which the largest is chosen, a function of Tb and the frequency of the 1 dof system chosen. At low frequency, it is better to choose Tb large enough to increase the number of peaks in each block, at high frequency, it is better to choose Tb small to obtain a large number of blocks and improve the choice of law.

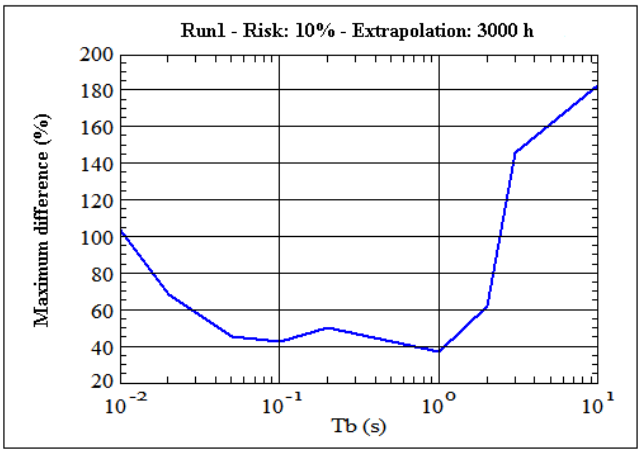

Figure 6. *Maximum difference between the DBM URS and the URS calculated from the PSD*

The calculation result of the DBM URS depends a lot on the Tb. In this example, there does not seem to be a value of Tb such that the difference with the URS from the PSD is less than 40%.

However, there is no criterion for choosing Tb, other than that the number of blocks be greater than 40. The justification for this number is also not provided in the standard.

One can wonder about the industrial use that can be made of DBM spectra without having a selection criterion for this parameter necessary for the reproducibility of results from several users.

3.4. Obtaining unrealistic values

To assess the consequences of the extrapolation of the duration, the largest peak of each of Nb blocks of duration Tb obtained for each dof during the DBM analysis was noted and calculated from all of these peaks the mean value + n standard deviations, without assumption on the law.

Figure 7 shows the spectrum thus calculated as well as the DBM URS 1% calculated for a duration of 10,000 h.

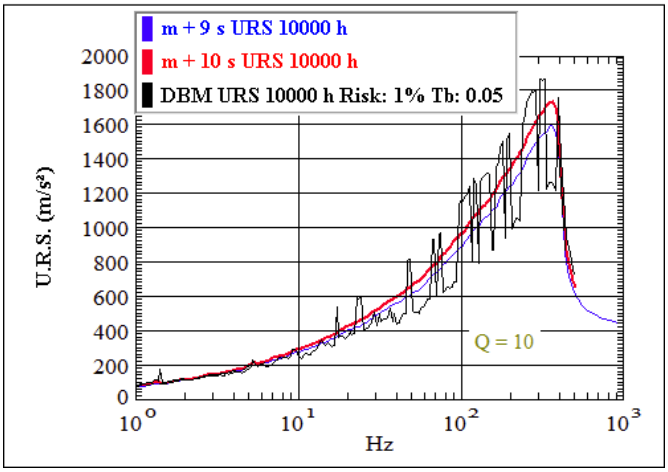

Figure 7. *Comparison of URS (Run1) extrapolated to 10000h vs an URS over the duration of the signal at 9 or 10 standard deviations*

We see that it takes about 9 to 10 standard deviations to reach the DBM URS, which corresponds to a probability of the order of 10^{-9}. This means that it is estimated that there could be statistically over this duration a peak in the response with this amplitude. The question of the physical reality of these peaks may arise.

A sporadic up-crossing of 9 or 10 standard deviations does not seem physically realistic. A physical process, whatever it is, is always limited by a finite energy [PIE 11]. Taking these values into account would probably lead to protection against peaks that will never occur.

3.5. Variability of URS obtained from several signals generated under the same conditions

DBM URSs are calculated for a given risk of up-crossing. From the same PSD, 24 signals were generated, each with a single inverse transform (no splicing). 24 DBM URSs were then calculated over a period of 10,000 h (Tb = 0.05, Risk = 10%), then their mean, their standard deviation (Figure 8) and the coefficient of variation (Figure 9). The max and min envelopes of the 24 spectra are plotted in Figure 10.

It is noted that the variability is significant and that the calculation of an URS on a single signal cannot make it possible to ensure that from a single measurement, that there will not be in the treated environment values greater than those calculated with the chosen risk.

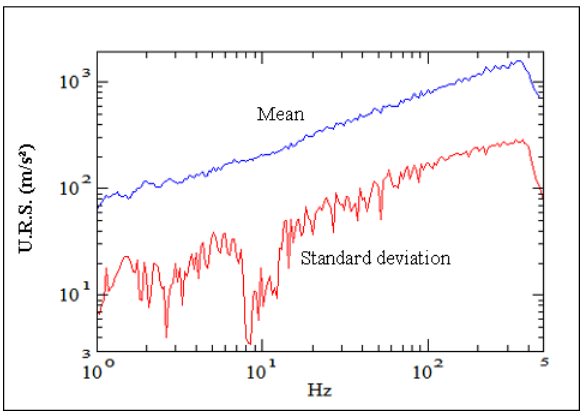

Figure 8. *Mean URS and Standard Deviation of the 24 signals*

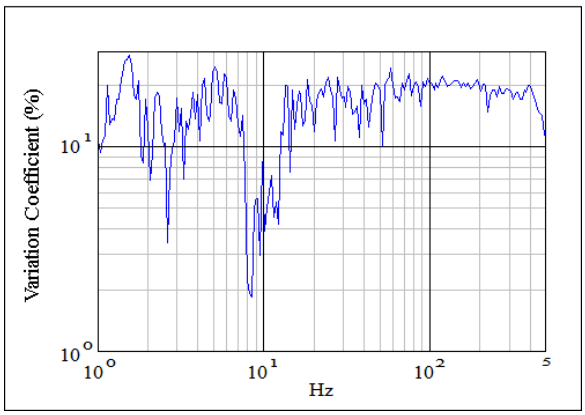

Figure 9. *Coefficient of variation of the URS of the 24 signals*

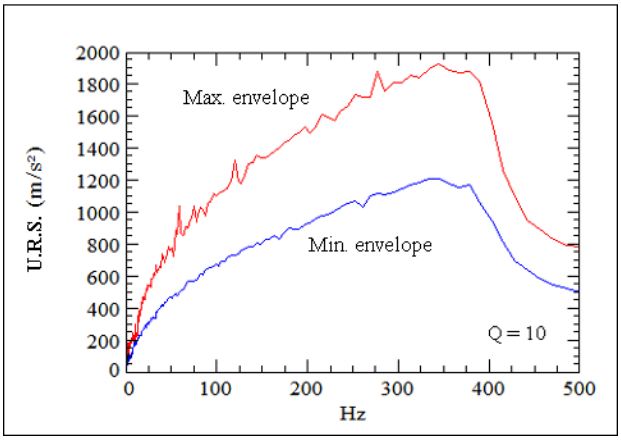

Figure 10. *URS envelopes of the 24 signals*

3.6. DBM eligible signals

The Standard does not specify which signals are actually eligible for DBM among those that are non-stationary and/or non-Gaussian. We already know that this is not the case for the following signals:

- case of strong shocks: the criteria for the “strong” level threshold remain to be specified,
- case of the single or multi-line swept sine plus noise: Appendix A of the Standard does not mention this and Appendices B and C do not mention the particular precautions to be taken, such as locking Tb on the frequency of the 1st line, so that the DBM can work,
- perhaps other particular cases of signals which remain to be discovered.

4. CONCLUSION

In summary, it was found that:

- the URS are tainted by fluctuations which appear more particularly for the low values of the risk and for the extrapolations of duration, the origin of which is not fully identified,
- there is a non-negligible variability on the URS calculated from several occurrences of a RUN type signal, so that the URS calculated from a single signal does not make it possible to ensure non-exceedance for a given risk , contrary to the intended purpose. One could hope to achieve this by applying a CVsup (if it were known, which is not the case) making it possible to calculate the product CG x Fe to be applied to the URS of a draw in order to obtain the URS covering the entire draw population,
- the DBM URS is sensitive to the block duration value chosen for the analysis. This duration should be long enough at low frequency so that there are enough peaks in the response and shorter at high frequency to improve the statistic (more blocks). However, no rule is proposed

despite the importance of this parameter (apart from a number Nb of blocks greater than 40, which is not enough),

- no resetting of the DBM for the case of a Gaussian signal with the classical method, and a fortiori of course with a non-Gaussian / non-stationary signal,

- the DBM method is not eligible for all types of signals:

The DBM presented in the Afnor Standard NF 50-144-3 has the relevant objective of making it possible to process non-stationary and/or non-Gaussian vibration signals. The evaluation of this method in the current state of the Standard shows that its use comes up against a certain number of difficulties, the most important of which are the non-connection with the classic method in the Gaussian case, the uncontrolled influence of the choice a priori of the duration of the blocks and the fluctuations present on the URS. Additional work is needed to try to make this method usable, although it seems interesting in principle.

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